1) $f(x) = \sqrt[4]{4-x^2} + \arcsin \lg(1-3x)$

$$\begin{cases} 4-x^2 \geq 0 & \text{i)} \\ -1 \leq \lg(1-3x) \leq 1 & \text{ii)} \\ 1-3x > 0 & \text{iii)} \end{cases}$$

i) $4-x^2 \geq 0 \iff -2 \leq x \leq 2$

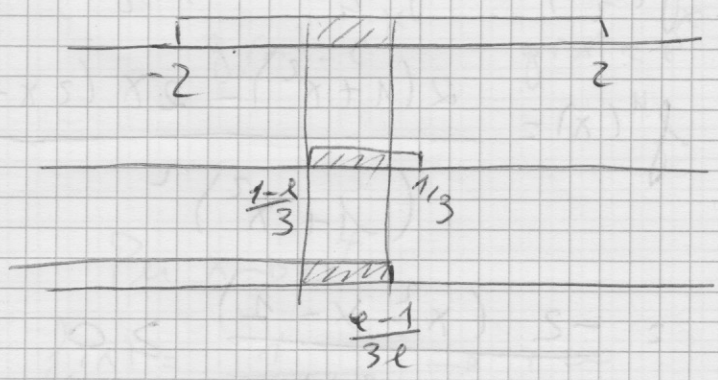
ii)-iii) $\lg(1-3x) \leq 1 \iff 0 < 1-3x \leq e \iff$

$\iff \frac{1-e}{3} \leq x < \frac{1}{3}$

$\lg(1-3x) \geq -1 \iff 1-3x \geq \frac{1}{e} \iff x \leq \frac{e-1}{3e}$

Il sistema è equivalente a

$$\begin{cases} -2 \leq x \leq 2 \\ \frac{1-e}{3} \leq x < \frac{1}{3} \\ x \leq \frac{e-1}{3e} \end{cases}$$



$$\frac{1-e}{3} \leq x \leq \frac{e-1}{3e}$$

2) $f(x) = \lg(1+x^2) - \operatorname{arctg} x$

CE $\mathbb{R}$ $\qquad x=0 \Rightarrow y=0$

Comportamento agli estremi

$\lim_{x\to+\infty} f(x) = +\infty$; $\lim_{x\to-\infty} f(x) = +\infty \Rightarrow$ No Asint. orizzontali

$$\lim_{x\to+\infty} \frac{\lg(1+x^2)-\operatorname{arctg} x}{x} \overset{(H)}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to+\infty} \frac{\frac{2x}{1+x^2}-\frac{1}{1+x^2}}{1} = 0$$

$$\lim_{x\to-\infty} \frac{\lg(1+x^2)-\operatorname{arctg} x}{x} = 0$$

Studio di crescente e decrescente

$$f'(x) = \frac{2x}{1+x^2} - \frac{1}{1+x^2} = \frac{2x-1}{1+x^2}$$ esiste $\forall x \in \mathbb{R}$

$f'(x) \geq 0 \iff 2x-1 \geq 0 \iff x \geq \frac{1}{2}$

f	$\searrow$		$\nearrow$
f'	$-$	$1/2$	$+$

$x = \frac{1}{2}$ punto di minimo rel (A.M.)

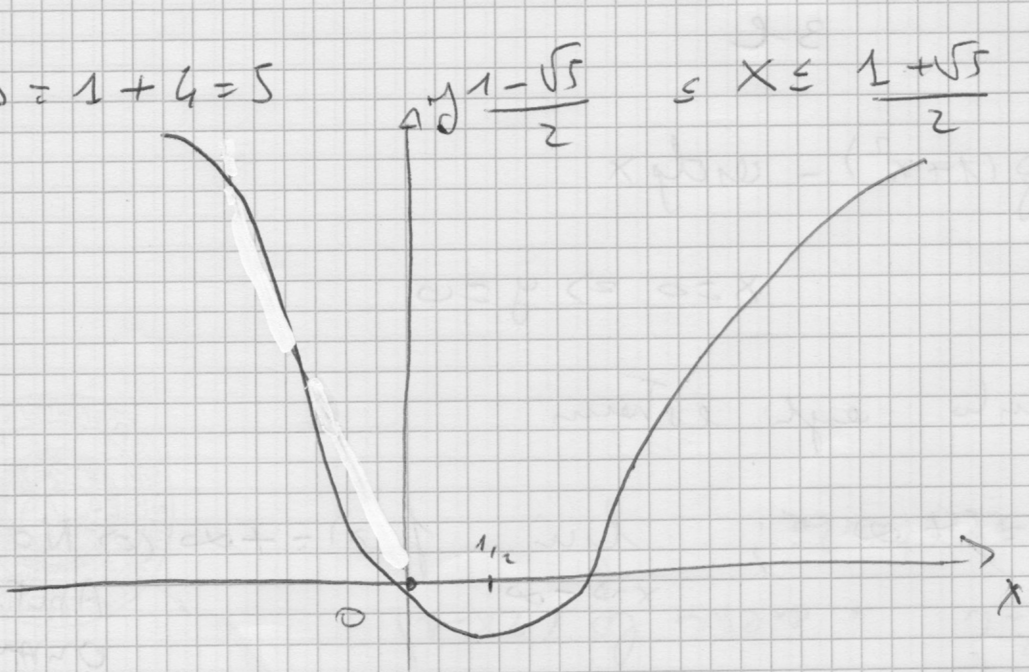
$f\left(\frac{1}{2}\right) = \lg \frac{5}{4} - \operatorname{arctg} 1/2 \cong -0{,}23$

$$f''(x) = \frac{2(1+x^2) - 2x(2x-1)}{(1+x^2)^2} = \frac{-2x^2+2x+2}{(1+x^2)^2} =$$

$$= \frac{-2(x^2-x-1)}{(1+x^2)^2} \geq 0 \iff x^2-x-1 \leq 0$$

$\Delta = 1+4 = 5$ $\quad \frac{1-\sqrt{5}}{2} \leq x \leq \frac{1+\sqrt{5}}{2}$

f''	$-$		$+$		$-$
f	$\cap$	$\frac{1-\sqrt{5}}{2}$	$\cup$	$\frac{1+\sqrt{5}}{2}$	$\cap$



3) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{e^{x^2}-1}{\lg(x^2+1)} = \frac{0}{0} = \lim_{x \to 0^+} \frac{e^{x^2} \cdot 2x}{\frac{2x}{1+x^2}} =$

$= \lim_{x \to 0^+} \frac{e^{x^2} \cdot \cancel{2x}}{\cancel{2x}} (1+x^2) = 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{2x^2+x^3}{(x-1)^2} = 0 = f(0)$

La f in $x=0$ presenta una discontinuità di I specie.
In $\mathbb{R} \setminus \{0\}$ f è continua.

4a) $\int (2x^2+1) e^x dx =$

$f(x) = 2x^2+1 \qquad f'(x) = 4x$
$g'(x) = e^x \qquad g(x) = e^x$

$f(x) = x \qquad f'(x) = 1$
$g'(x) = e^x \qquad g(x) = e^x$

$= (2x^2+1)e^x - 4\int x e^x dx =$

$= (2x^2+1)e^x - 4\left[x e^x - \int e^x dx\right] =$

$= (2x^2+1)e^x - 4xe^x - 4e^x + C$

5) $\int_0^1 \frac{1}{\sqrt{x}(1+\sqrt{x})^3} dx$

Per $x \to 0^+$
$f(x) = \frac{1}{\sqrt{x}(1+\sqrt{x})^3} \to +\infty$

$\lim_{x \to 0^+} \frac{\frac{1}{\sqrt{x}(1+\sqrt{x})^3}}{\frac{1}{\sqrt{x}}} = 1$

Poiché $\frac{1}{\sqrt{x}}$ è integrabile in senso improprio in $(0,1)$ $\Rightarrow$ anche $f(x) = \frac{1}{\sqrt{x}(1+\sqrt{x})^3}$ è integrabile in senso improprio in $(0,1)$.

$$\int_0^1 \frac{1}{\sqrt{x}\,(1+\sqrt{x})^3}\,dx =$$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$x = \varepsilon \Rightarrow t = \sqrt{\varepsilon}$

$x = 1 \Rightarrow t = 1$

$$= \lim_{\varepsilon \to 0^+} \int_{\varepsilon}^{1} \frac{1}{\sqrt{x}\,(1+\sqrt{x})^3}\,dx$$

$$= \lim_{\varepsilon \to 0^+} \int_{\sqrt{\varepsilon}}^{1} \frac{1}{t\,(1+t)^3}\,2\cancel{t}\,dt = 2 \lim_{\varepsilon \to 0^+} \left[\frac{(1+t)^{-2}}{-2} \right]_{\sqrt{\varepsilon}}^{1}$$

$$= 2 \lim_{\varepsilon \to 0^+} \left(-\frac{1}{2}\right) \left[\frac{1}{(2)^2} - \frac{1}{(1+\sqrt{\varepsilon})^2} \right] = -\left[\frac{1}{4} - 1\right] = \frac{3}{4}$$

5) $$\sum_{n=1}^{\infty} \frac{(\lg 2)^n}{2n}$$

Serie a termini positivi

$$\lim_{n \to +\infty} \sqrt[n]{\frac{(\lg 2)^n}{2n}} = \lim_{n \to +\infty} \frac{\lg 2}{\underbrace{\sqrt[n]{2n}}_{\to 1}} = \lg 2 < 1$$

La serie è convergente

6) $z^2 + 4(1+\sqrt{3}i) = 0$

$$z = \sqrt{-4-4\sqrt{3}i}$$

$$\rho = \sqrt{16+16\cdot 3} = \sqrt{64} = 8$$

$$\begin{cases} \cos\vartheta = -\frac{1}{2} \\ \sin\vartheta = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \quad \vartheta = -\frac{2}{3}\pi$$

$$w_k = \sqrt{8}\left[\cos\frac{-\frac{2}{3}\pi + 2k\pi}{2} + i\sin\frac{-\frac{2}{3}+2k\pi}{2}\right]$$

$k = 0, 1$

$$w_0 = 2\sqrt{2}\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right] =$$

$$= 2\sqrt{2}\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = \sqrt{2} - i\sqrt{6}$$

$$w_1 = 2\sqrt{2}\left[\cos\left(\frac{2}{3}\pi\right) + i\sin\left(\frac{2}{3}\pi\right)\right] = 2\sqrt{2}\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$$

$$= -\sqrt{2} + i\sqrt{6}$$