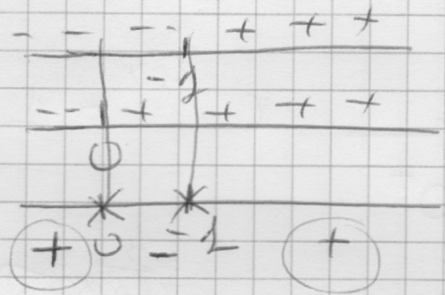
1) $f(x) = \arctg \lg \frac{x+1}{x} + \sqrt{x^2-3x+2} + \arcsin e^x$

$$\begin{cases} \frac{x+1}{x} > 0 \\ x^2-3x+2 \geq 0 \\ -1 \leq e^x \leq 1 \end{cases} \quad (*)$$

i) $\frac{x+1}{x} > 0$ $\quad N > 0 \quad x+1>0$

$\quad D > 0 \quad x > 0$



$\iff x < -1 \,;\, x > 0$

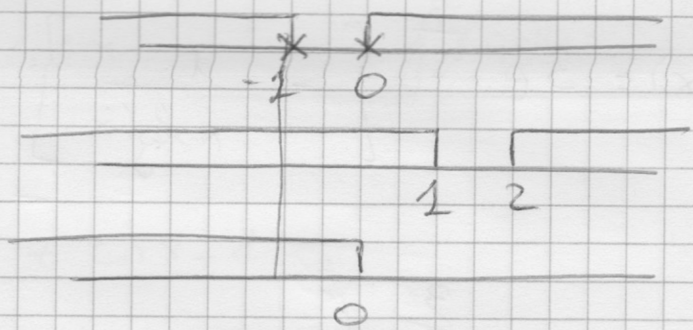
ii) $x^2-3x+2 \geq 0 \quad \Delta = 9-8 = 1 \quad x = \frac{3 \mp 1}{2} = \begin{smallmatrix} 1 \\ 2 \end{smallmatrix}$

$\iff x \leq 1 \,;\, x \geq 2$

iii) $e^x \geq -1 \quad \forall x \in \mathbb{R}, \quad e^x \leq 1 \iff x \leq 0$

Sistema $(*)$ è equivalente a

$$\begin{cases} x < -1 \,;\, x > 0 \\ x \leq 1 \,;\, x \geq 2 \\ x \leq 0 \end{cases}$$



$\iff x < -1$

2) $f(x) = e^{\frac{3}{\lg x}}$

C.E. $\begin{cases} x > 0 \\ \lg x \neq 0 \end{cases} \iff \boxed{x > 0, \; x \neq 1}$

$f(x) > 0 \quad \forall x \in CE$, nessuna intersezione con gli assi

Asintoti

$$\lim_{x \to +\infty} e^{\frac{3}{\lg x}} = 1$$

$x = 1$ As. orizzontale destro

$$\lim_{x \to 0^+} e^{\frac{3}{\lg x}} = 1$$

$\lg x \to 0^-$

$$\lim_{x \to 1^-} e^{\frac{3}{\lg x}} = 0$$

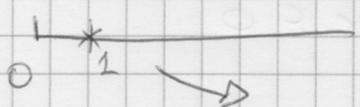
$(\lg x) \to 0^+$

$$\lim_{x \to 1^+} e^{\frac{3}{(\lg x)}} = +\infty$$

$x = 1$ As. verticale destro

Crescenza e decrescenza

$$f'(x) = 3 e^{\frac{3}{\lg x}} \left[-\frac{1}{x \lg^2 x} \right] < 0 \quad \forall x \in CE$$



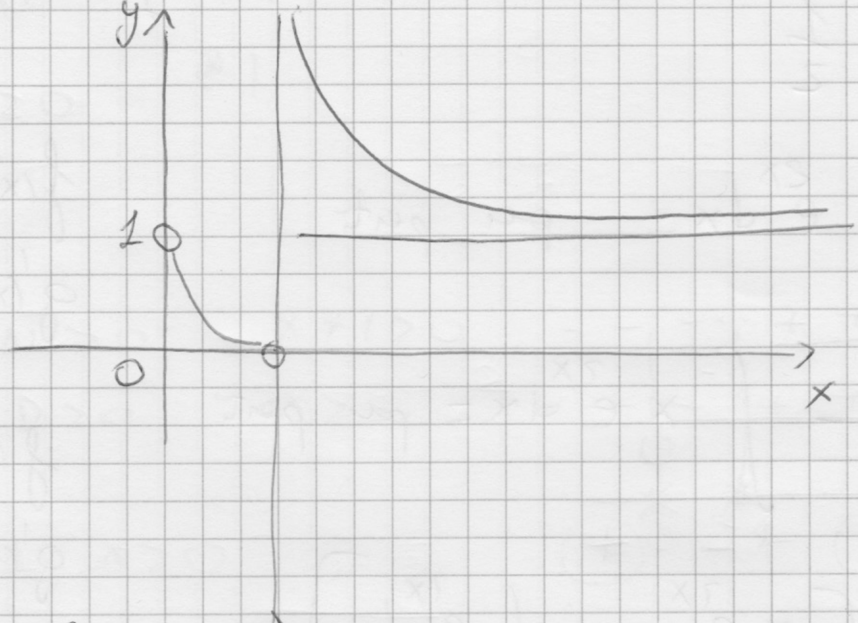
$$f''(x) = -3\, e^{\frac{3}{\lg x}} \left[-\frac{3}{x \lg^2 x} \left(-\frac{1}{x \lg^2 x} \right) - \frac{\lg^2 x + 2 \lg x \cdot \frac{x}{x}}{x^2 \lg^4 x} \right]$$

$$= +3\, e^{\frac{3}{\lg x}} \left[\frac{\lg^2 x + 2 \lg x + 3}{x^2 \lg^4 x} \right]$$

$$f''(x) \geq 0 \iff \lg^2 x + 2 \lg x + 3 \geq 0$$

$\frac{\Delta}{4} = 1 - 3 = -2 < 0$

$f''(x) \geq 0 \quad \forall x \in C\mathcal{E}$



3) $\lim\limits_{x \to 0} \dfrac{\lg(1+x^2) + \cos x - 1}{x^2}$

$f(x) = \lg(1+x^2) + \cos x - 1 \qquad f(0) = 0$

$f'(x) = \dfrac{1}{1+x^2} \cdot 2x - \sin x \qquad f'(0) = 0$

$f''(x) = \dfrac{2(1+x^2) - 4x^2}{(1+x^2)} - \cos x \qquad f''(0) = 2 - 1 = 1$

$\lim\limits_{x \to 0} \dfrac{\lg(1+x^2) + \cos x - 1}{x^2} =$

$= \lim\limits_{x \to 0} \dfrac{\frac{x^2}{2} + o(x^2)}{x^2} = \dfrac{1}{2}$

Svolgimento con il Teorema di De L'Hôpital

$\lim\limits_{x \to 0} \dfrac{\lg(1+x^2) + \cos x - 1}{x^2} = \dfrac{0}{0} = \lim\limits_{x \to 0} \dfrac{\frac{1}{1+x^2} \cdot 2x - \sin x}{2x} =$

$$= \lim_{x \to 0} \left(\frac{1}{1+x^2} \frac{\cancel{2x}}{\cancel{2x}} - \frac{1}{2} \underbrace{\left(\frac{\sin x}{x}\right)}_{\downarrow\, 1} \right) = 1 - \frac{1}{2} =$$

$$= \frac{1}{2}$$

44) $\int x^2 e^{2x} dx =$ per parti

$f(x) = x^2$ $\quad f'(x) = 2x$

$g'(x) = e^{2x}$ $\quad g(x) = \frac{e^{2x}}{2}$

$$= \frac{x^2 e^{2x}}{2} - \int x e^{2x} dx = \text{per parti}$$

$f(x) = x$ $\quad f'(x) = 1$

$g'(x) = e^{2x}$ $\quad g(x) = \frac{e^{2x}}{2}$

$$= \frac{x^2 e^{2x}}{2} - \left[\frac{x e^{2x}}{2} - \frac{1}{2} \int e^{2x} dx \right] =$$

$$= \frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{1}{4} e^{2x} + C$$

45) $\int_0^{+\infty} \frac{x-1}{(x^2-2x+2)^2} dx$

$x^2 - 2x + 2 = 0$ $\quad \nexists$ in $\mathbb{R}$

$\frac{\Delta}{4} = 1 - 2 = -1 < 0$

$$\lim_{x \to +\infty} \frac{\frac{x-1}{(x^2-2x+2)^2}}{\frac{1}{x^3}} = 1$$

Per il criterio asintotico, poiché $\frac{1}{x^3}$ è integrabile in $(1, +\infty)$, anche $f(x) = \frac{x-1}{(x^2-2x+2)}$ è integrabile in $(1, +\infty)$

Calcolo dell'integrale

$$\int_0^{+\infty} \frac{x-1}{(x^2-2x+2)^2} dx = \lim_{R \to +\infty} \int_0^R \frac{x-1}{(x^2-2x+2)^2} dx =$$

$$= \lim_{R \to +\infty} \frac{1}{2} \int_0^R \frac{2x-2}{(x^2-2x+2)^2}\, dx =$$

$$= \lim_{R \to +\infty} \frac{1}{2} \left[\frac{(x^2-2x+2)^{-1}}{-1} \right]_0^R = \frac{1}{2} \ell$$

$$= \frac{1}{2} \lim_{R \to +\infty} \left[-\frac{1}{R^2-2R+2} + \frac{1}{2} \right] = \frac{1}{4}$$

5) $$\sum_{n=1}^{+\infty} \frac{5^n}{n \cdot n!}$$

Poiché è una serie a termini non negativi, possiamo applicare il criterio del rapporto

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = \lim_{n \to +\infty} \frac{\dfrac{5^{n+1}}{(n+1)(n+1)!}}{\dfrac{5^n}{n \cdot n!}} =$$

$$= \lim_{n \to +\infty} \frac{\cancel{5^n} \cdot 5}{(n+1)(n+1) \cdot \cancel{n!}} \cdot \frac{\cancel{n!}\, n}{\cancel{5^n}} = \lim_{n \to +\infty} 5 \frac{n}{(n+1)^2} =$$

$= 0 < 1 \Rightarrow$ la serie $\sum_{n=1}^{+\infty} \frac{5^n}{n \cdot n!}$ converge

6) $$z^4 + 32(1+\sqrt{3}\, i) = 0$$

$$\sqrt[4]{-32(1+\sqrt{3}\, i)} =$$

$$= 2 \sqrt[4]{-2-2\sqrt{3}\, i}$$

$z = -2 - 2\sqrt{3}\, i$

~~$\rho =$~~ $\rho = \sqrt{4 + 4 \cdot 3} = \sqrt{16} = 4$

$$\begin{cases} \cos\theta = -\frac{1}{2} \\ \sin\theta = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \theta = -\frac{2}{3}\pi$$

$$z = -2-2\sqrt{3}\,i = 4\left[\cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)\right]$$

$$\sqrt[4]{-2-2\sqrt{3}\,i}$$

$$w_k = \sqrt[4]{4}\left[\cos\frac{-\frac{2}{3}+2k\pi}{4} + i\sin\frac{-\frac{2}{3}+2k\pi}{4}\right]$$

$k = 0,1,2,3$

$$w_0 = \sqrt{2}\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right] =$$

$$= \sqrt{2}\left[+\frac{\sqrt{3}}{2} - \frac{i}{2}\right]$$

$$w_1 = \sqrt{2}\left[\cos\left(\frac{\pi}{3}\right) + i\sin\frac{\pi}{3}\right] = \sqrt{2}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$$

$$w_2 = \sqrt{2}\left[\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right] = \sqrt{2}\left[-\frac{\sqrt{3}}{2} + \frac{i}{2}\right]$$

$$w_3 = \sqrt{2}\left[\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right] = \sqrt{2}\,\cancel{\cos}\left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$$

Quindi $\sqrt[4]{-32(1+\sqrt{3}\,i)} = 2\sqrt[4]{-2-2\sqrt{3}\,i}$ sono

$$z_0 = \sqrt{2}\left[\sqrt{3} - i\right]$$

$$z_1 = \sqrt{2}\left[1 + i\sqrt{3}\right]$$

$$z_2 = \sqrt{2}\left[-\sqrt{3} + i\right]$$

$$z_3 = \sqrt{2}\left[-1 - i\sqrt{3}\right]$$